

Holography in a quantum spacetime

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Abstract

We propose a formulation of the holographic principle, suitable for a background independent quantum theory of cosmology. It is stated as a relationship between the flow of quantum information and the causal structure of a quantum spacetime. Screens are defined as sets of events at which the observables of a holographic cosmological theory may be measured, and such that information may flow across them in two directions. A discrete background independent holographic theory may be formulated in terms of information flowing in a causal network of such screens. Geometry is introduced by *defining* the area of a screen to be a measure of its capacity as a channel of quantum information from its null past to its null future. We call this a “weak” form of the holographic principle, as no use is made of a bulk theory.

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1 Introduction

In this paper, we present a framework for a Planck-scale, cosmological, background-independent theory which is holographic in a sense appropriate to a quantum spacetime. This is motivated by the fact that the formulations of the holographic principle given to date[1]-[6] are confined to the semiclassical regime. At the same time, results of several approaches to quantum gravity indicate that the description of spacetime based on smooth manifolds can provide only an approximate description[7]-[12]. If it is true, the holographic principle ought then to be more than a conjecture about the classical and semiclassical theory. Rather, it should be an important part of the framework of a Planck-scale, background independent quantum theory.

Our goal is to give a form of the holographic principle that could be satisfied by a background independent quantum theory of gravity, and reduces to the standard holographic principle in the semiclassical limit. To guide us, we make two assumptions. First, the theory must be cosmological, in the sense that whatever structure replaces the smooth spacetime geometry will have no fixed external boundary or asymptotic regions. This means that we must keep in mind what has been learned about how to formulate the holographic principle at the semiclassical level in cosmological spacetimes. One important lesson, discovered first by Fischler and Susskind[3] and developed by Bousso[4], is that in a cosmological spacetime the holographic principle must be formulated in a way that employs the lightcones of the spacetime. Otherwise, paradoxes arise which are discussed in [3, 4, 6]. Some of these paradoxes concern cases in which the number of bulk degrees of freedom associated with non-null surfaces fails to be bounded, or even fails to be well defined. These same problems arise when we attempt to formulate a version of the holographic principle suitable for a background independent theory[13]. To avoid them, we assume here that there is a fundamental causal structure which plays a role in the background independent theory and that in the classical limit this becomes the causal structure of a spacetime. This is our second assumption.

To proceed, we need a framework for a cosmological quantum theory which incorporates causal structure. We require that this framework be sufficiently general that it can accomodate a background independent quantum theory of gravity. Such a framework, called quantum causal histories, was defined in [14]. It is cosmological in the sense that any physically meaningful observable corresponds to some observer (represented as an event or collection of events) inside the closed universe. (In the classical case, see [15]). As each observer receives information from a distinct past, the algebra of observables they can measure, and hence the (finite-dimensional) Hilbert spaces on which what they observe can be represented, vary over the history. Consequently, the algebra of observables of the theory is represented on a collection of Hilbert spaces. These replace the single wavefunction and single Hilbert space of other approaches to quantum cosmology.

Quantum causal histories were originally motivated by the need to provide a

general framework to understand what observables are in background independent approaches to quantum gravity, such as those proposed in [16, 18, 19, 20]. In these theories a causal quantum spacetime is constructed from local changes in a spin network or a network of abstract surfaces. The Hilbert spaces used to describe such changes are finite-dimensional because only a finite amount of information about the quantum geometry is involved in each elementary causal process. As one expects from a scheme of this type, in which a fundamental discreteness in the structure of spacetime appears at Planck scales, a key question the theory must answer is whether a smooth spacetime geometry is recovered in the continuum limit.

The question we address in this paper is what properties such a theory must have in order that its semiclassical limits, when they exist, are holographic in the sense defined in [1, 2, 3, 4, 6]. A central element in the semiclassical formulations of the cosmological holographic principle is that of a screen, a spacelike 2-surface on which the relevant degrees of freedom of the theory live¹[4]. We require that there are analogues of these 2-surfaces in the quantum spacetime. In the next section we define what we call an “elementary screen”, these are certain collections of events in a quantum causal history. We then define a class of quantum spacetimes consisting of causal networks of such screens. We call these *screen networks*. We show in section 4 that there are examples of them which may be constructed by imposing certain restrictions on the class of background independent theories of quantum gravity and string theory given in [16, 18, 19, 20].

An obvious fact about 2-dimensional surfaces turns out to be key in this work, namely, that each 2-surface has two sides. We find below that this has two important consequences. First, the additional structure in a screen network which follows from the two-sided nature of the elementary screens allows a distinction between null and timelike propagation, something that an ordinary causal set history does not provide. Second, this makes it possible to incorporate chirally asymmetric theories in the causal quantum history framework.

An essential element of the holographic principle in its semiclassical forms is the Bekenstein bound[21]. This must be recovered in the semiclassical limit of any holographic theory. This seems to present a potential problem, as metric plays no role in the definition of a screen network. Given that the metric is unified with other degrees of freedom in string theory, it is not even clear that the notion of area should be well defined at the background independent level. The only property a screen has beyond its place in the causal network is the dimension of its Hilbert space. This leads us to suggest that the Bekenstein bound may be inverted and *area be defined to be a measure of the capacity of a screen for the transmission of quantum information*.

The result is a form of the holographic principle which makes no use at all of the notion of a bulk theory, and instead posits only a relationship between the information capacity and the geometrical area of a screen. We call this a “weak holographic

¹In dimensions other than 4, a screen is a spacelike surface of codimension 2 in the spacetime.

principle”, and reserve “strong” for those formulations of the principle that posit relationships between bulk and boundary theories, or limits on the amount of information on spacelike or null surfaces bounded by the screens. We believe that strong forms of the principle are only relevant in the semiclassical theory, when the bound is on the number of matter degrees of freedom in a region of a single fixed spacetime and that, once the gravitational degrees of freedom are introduced, either classically or quantum mechanically, only weak forms of the principle are possible.

A key feature of the weak holographic principle is that a complete description of the universe requires more than one screen. This is simply because in a generic cosmological history there is no single screen whose past is the entire universe. Thus, a cosmological holographic theory must be a many-screens theory, each screen recording information about its causal past. It is important to note that such a many-screens theory gives us the possibility to dispense with the notion of the bulk theory. Rather than formulate the holographic principle in terms of a relationship between a bulk theory and a boundary, as is done in its strong forms, we can formulate it entirely in terms of screen observables and relationships between them.

A question left open for future work is the exact relationship between weak and strong forms of the holographic principle. It may be conjectured that when the weak holographic principle applies to a quantum causal history which has a good classical limit, the strong holographic principle holds in that limit. We expect that, when the limit exists, 2-dimensional surfaces in the continuum spacetime will originate from ensembles of elementary screens and that their area will satisfy the Bekenstein bound in the standard way. At the same time, we expect that no cosmological form of the holographic principle, even one that holds in the semiclassical limit, can escape the fact that many screens are necessary to give a complete description of a cosmological spacetime.

The outline of this paper is as follows. In the next section we define screens and screen networks. In section 3, a quantum screen network is defined, following the general prescription of [14], as a functor from the edge-sets of a screen network to the Hilbert space category. In section 4 we show what restrictions are imposed on the background independent formulations of quantum gravity and string theory given in [16, 18, 19, 20] if one requires that all events are elementary screens. In section 4 the essential features of this approach to the holographic principle are then abstracted and formulated as a proposal for the *weak holographic principle*. Comments on the correspondence to the semiclassical holographic principle as well as on the conditions required, were a strong form of the holographic principle to hold in a background independent theory, can be found in the final section.

2 Elementary screens and screen networks

We begin by recalling the definition of a causal set, as used by Sorkin, 't Hooft and others to describe the discrete analogue of the causal relations of events in a Lorentzian spacetime (see [23, 25, 24, 26]). Then we explain what causal histories are, and define elementary screens and screen networks.

A causal set \mathcal{C} is a locally finite, partially ordered set of events. That is, if we denote the events by p, q, r, \dots and, say, p precedes q , we write $p \leq q$. The equal option is used when p coincides with q . The causal relation is reflexive, i.e. $p \leq p$ for every event p and transitive, i.e. if $p \leq q$ and $q \leq r$, then $p \leq r$. It is also antisymmetric, that is, if $p \leq q$ and $q \leq p$, then $p = q$, which ensures that there are no closed timelike loops in the causal set. Local finiteness means that, given $p \leq q$, there is a finite number of events which are both in the future of p and in the past of q .

A causal relation $p \leq q$ is called an “edge” or a “covering relation” if it is not implied by transitivity from other relations in the history.

In our previous work, we have used the term “causal history” to describe causal sets whose events carry additional structure. For example, in causal histories of spin networks [16, 18, 19] the events are local changes in spin networks, while in [19] they are local changes in (p, q) string networks. For the reasons we explained in the introduction, in this paper we are concerned with histories whose events are *elementary screens*. An elementary screen s is a quadruple

$$s \equiv \{s_L^-, s_R^-, s_L^+, s_R^+\}. \quad (1)$$

It has four components, the past left, s_L^- , past right, s_R^- , future left, s_L^+ and future right s_R^+ . Within each quadruple there are maps from the past left component to the future right one, and from the past right to the future left, namely,

$$\begin{aligned} LR: s_L^- &\rightarrow s_R^+, \\ RL: s_R^- &\rightarrow s_L^+. \end{aligned} \quad (2)$$

These maps are the contribution of the screen s to the dynamics of the causal history.

A special case of a causal history is when all the events are elementary screens. We will call such a history a *screen network*. It is a partially ordered set of screens, in which two screens are related, $s \leq t$, when one of the future components of s precedes one of the past components of t . The following condition is imposed on a screen network: There can be at most one edge (covering relation) from s to t . This means that, if s is in the immediate past of t , t can only “see” one side of s .

Thus, the network describes signals exchanged amongst a set of elementary surfaces that make up a quantum version of a spacetime. We have called the two sides of a screen L and R , and each has a future and a past. According to (2), information

that comes into the past of the left side of a screen in the network may exit only from the right side of a screen, and vice versa. Thus, information is carried from one side of the screen to the other by the internal maps LR and RL . Components of different screens are related by the external (to the screen) maps \leq . These relations encode the causal structure of the screen network.

Having defined the screen network, the following sets can be constructed from it and will be used in the remaining of this paper.

- The causal past, $P(s)$, of a screen s is the set of screens t in the screen network with $t \leq s$.
- The left null past, LNP , of a component of a screen s is the set of those screens in its causal past that are related to s by a sequence of alternating LR and \leq maps. Its right null past, RNP , is the set of those past screens that are related to s by a sequence of alternating RL and \leq maps. The null past of s is the union of these two sets.
- The timelike past $TP(s)$ of a screen s is the screens in its causal past that are not null related to s , that is, $TP(s) = P(s) - NP(s)$.
- The causal future $F(s)$, null future $NF(s)$ and timelike future $TF(s)$ of s are similarly defined.

It is interesting to note that the two sides of a screen (the two internal maps) allow this natural distinction between null and timelike. Of course, for a general screen network, there is no global decomposition into left and right flows of information. Still, the two-sided nature of the screens gives the elementary processes and the flow of information in the network a chiral aspect, as left and right flows can always be distinguished locally.

A screen network can be reduced to its underlying causal set by removing the internal LR and RL maps and compressing the four screen components to a single causal set event.

3 The quantum screen network

We next wish to turn a screen network S into a network of elementary quantum-mechanical systems. In doing so, we will assume that quantum information propagates without change between screens and undergoes non-trivial evolution only when going through a screen. We express this by assigning a Hilbert space to every edge of the screen network, and two (unitary) evolution operators to each screen.²

²Another possibility is to do the reverse, namely, turn each screen into a finite-dimensional Hilbert space and each edge into an evolution map. However, as it was discussed in [14], as soon as this

Before we give the definition of the quantum screen network³, we list the two of its desired features that serve as the starting point. First, we wish to replicate the fact that, in quantum mechanics, the composite state space of spacelike separated systems is the tensor product of the individual state spaces. The individual systems in the screen network case are the edges e_i connecting different screens. Each edge is represented by a finite-dimensional state space $H(e_i)$. Two such edges are spacelike separated when there is no null or timelike path from the one to the other. Thus, given a set of spacelike separated edges, $a = \{e_1, e_2, \dots, e_n\}$, the composite state space is $H(a) = H(e_1) \otimes H(e_2) \otimes \dots \otimes H(e_n)$.

Second, if such a set of edges a is in the past of another set b , we can only expect to have a unitary evolution map from $H(a)$ to $H(b)$ if there has been no “loss” or “gain” of information from a to b . What this means for a screen network is the following. Consider two edge-sets a and b containing no common edges. Let every edge in a be in the past of some edge in b . Furthermore, let every edge in b be in the future of some edge in a . Then, in the notation of [14], a and b are a *complete pair*. Since there are no edges in the future of a that are spacelike to b and no edges in the past of b that are spacelike to a , a complete pair serves as a model of information conservation. In a quantum screen network, we expect to have unitary evolution only between edge sets that are complete pairs.

Keeping the above in mind, we define the *edge screen network*, ES , to be the partially ordered set whose elements are edge-sets, sets of spacelike separated edges a, b, \dots in the screen network S . Two edge-sets are related in ES when they are a complete pair.

We may now define the quantum screen network as a functor from the edge screen network to the category of Hilbert spaces (which has Hilbert spaces for its objects and unitary operators as its arrows). Hence, a quantum screen network QS , is the functor

$$QS : ES \rightarrow Hilb, \quad (3)$$

such that for every edge-set a in ES there is a finite-dimensional Hilbert space $H(a)$ in QS . If a and a' are spacelike separated (have no common edges), $H(a \cup a') = H(a) \otimes H(a')$. For every complete pair $a \leq b$ in ES , $\dim H(a) = \dim H(b)$, and there is a unitary evolution operator $E : H(a) \longrightarrow H(b)$ in QS .

According to the above, for some screen s in the screen network, $H(s_L^-)$ is the state space of the edges going into the left of the screen, and $H(s_R^-)$ the state space of the edges going into the right of the screen. A screen has the same information

is done, acausal evolution becomes possible and the quantum mechanical information flow does not reflect the underlying causal set anymore. The solution that was proposed in [14] is the recipe used here, i.e. attach the Hilbert spaces on the edges and the operators to the events. This is well-motivated physically as it agrees with the intuition that events should represent change, and so their quantum-mechanical counterpart should be an operator rather than a state space.

³We may note that this differs from the quantum causal sets defined by Criscuolo and Waelbroeck[22].

capacity on both sides, which implies that

$$\dim H(s_L^-) = \dim H(s_R^-). \quad (4)$$

Clearly, s_L^- and s_R^+ is a complete pair, and so is s_R^- and s_L^+ . The unitary operators in QS corresponding to these two complete pairs are $\widehat{LR}(s)$ and $\widehat{RL}(s)$. By the unitarity of the operators in QS we have

$$\dim H(s_L^-) = \dim H(s_R^+) \quad \text{and} \quad \dim H(s_R^-) = \dim H(s_L^+). \quad (5)$$

Thus, the state spaces of all the components of a screen s have the same dimension, which we denote $D(s)$.

We define the area of a screen s to be a measure of the information capacity of a screen. This is proportional to the dimension of the Hilbert space of any of the components of s :

$$A(s) \equiv a l_{Planck}^2 \ln D(s). \quad (6)$$

where a is a constant, which we may take to be equal to $1/4$ to agree with the semiclassical Bekenstein bound. Finally, any evolution operator $E_{ab} : H(a) \rightarrow H(b)$ in QS can be decomposed into \widehat{RL} and \widehat{LR} operators in the screens between a and b .

We claim that a quantum screen network is a holographic theory because the generating evolution operators \widehat{RL} and \widehat{LR} act on Hilbert spaces on one side of some screen. Simply because a screen has two sides, we regard it as the quantum spacetime analogue of a spacetime object of codimension 2.

4 Causal spin network evolution

We turn now to the question of what restrictions may be imposed on candidates for background independent quantum theories of gravity by the requirement that the corresponding quantum causal histories are screen networks. We consider here the example of causal histories of spin networks, in the original form given in [16], or the extensions in [18, 19, 20]. We first review why the histories in such theories are quantum causal histories[14], and then ask what additional conditions must be satisfied to ensure that they are screen networks.

In these histories, the analogue of a spatial region in a spacetime is an open spin network γ . This is an oriented graph (or, in [18, 19, 20], a punctured two-dimensional surface) with free ends whose edges are labeled by representations of a quantum group or supergroup. In the case of quantum general relativity this is taken to be $SU_q(2)$. Extensions to supergravity[27] or other dimensions are described by different quantum groups. We denote the labelled edges by e_i, e_j , etc.

An important observation is that γ labels a state in the space of intertwiners $\mathcal{V}_{\{e_i\}}$ of the representations labeling its free edges. The dimension of $\mathcal{V}_{\{e_i\}}$, given the labels on the free edges, can be calculated using the Verlinde formula in [28].

The open labelled graph γ is generally a piece of a closed spin network Γ which defines the quantum geometry of a complete spacelike slice of a spacetime history. See [16, 18] for details.

A local evolution move replaces γ with a new open graph, γ' , which has the same free ends $\{e_i\}$ as γ . The result is a bubble evolution move, in which only the local region γ evolves, leaving unchanged the remaining $(\Gamma - \gamma)$. As a result, the history is a quantum version of many-fingered time evolution.

By construction, γ and γ' have the same labeled free edges and therefore live in the same space of intertwiners $\mathcal{V}_{\{e_i\}}$. The move which replaces γ by γ' is then represented by a transition in the Hilbert space, $\mathcal{V}_{\{e_i\}}$. The dynamics of the theory can then be given by a rule which assigns an evolution operator to each such space of intertwiners. In this case,

$$\hat{E} : \mathcal{V}_{\{e_i\}} \longrightarrow \mathcal{V}_{\{e_i\}}. \quad (7)$$

To ensure that there is no loss of information *locally*, these operators are required to be unitary. All the generating evolution moves listed in [16], the so-called Pachner moves for abstract spin networks, are operators of this type.

This shows that to each causal spin network history \mathcal{M} , as described in [16, 18, 19, 20], there corresponds a quantum causal history $Q\mathcal{M}$. Each open spin network piece γ in \mathcal{M} is a Hilbert space $\mathcal{V}_{\{e_i\}}$ in $Q\mathcal{M}$. Every time there is an evolution move γ to γ' in \mathcal{M} , γ and γ' are a complete pair. γ' lives in the same Hilbert space $\mathcal{V}_{\{e_i\}}$ as γ and thus the move is a unitary operator in $Q\mathcal{M}$.

Now we turn to the additional requirements that arise if we want each such move to correspond to an elementary screen. This requires that we do two things. First, in each transition it must be possible to pick out four sets of edges, corresponding to the four components of a screen. Second, Hilbert spaces must be associated to them in such a way that the evolution splits into two parts according to eq. (2).

To accomplish this we note that the space of intertwiners $\mathcal{V}_{\{e_i\}}$ can be split as follows. We divide the external edges e_i into two sets, which we will call the left set e_L and the right set e_R . We may then write

$$\mathcal{V}_{\{e_i\}} = \bigoplus_j \mathcal{V}_{\{e_L\}j} \otimes \mathcal{V}_{\bar{j}\{e_R\}} \quad (8)$$

where j is short for e_j , the the sum is over a complete set j of the representations and \bar{j} is the complex conjugate representation.

We require that this choice be made so that for at least one j ,

$$\dim \mathcal{V}_{\{e_L\}j} = \dim \mathcal{V}_{\bar{j}\{e_R\}}. \quad (9)$$

We then pick a particular $j = j_0$ that satisfies this and restrict γ and γ' to lie in the subspace $\mathcal{V}_{\{e_L\}j_0} \otimes \mathcal{V}_{\bar{j}_0\{e_R\}}$. The evolution operator \hat{E} is then required to be of the form,

$$\hat{E} = \begin{pmatrix} 0 & \widehat{LR} \\ \widehat{RL} & 0 \end{pmatrix}. \quad (10)$$

This agrees with the general form implied by the application of eq.(3) to eq.(2) if

$$H(s_L^\pm) = \mathcal{V}_{\{e_L\}j_0} \quad (11)$$

and

$$H(s_R^\pm) = \mathcal{V}_{\{e_R\}\bar{j}_0}. \quad (12)$$

The two restrictions (9) and (10) are non-trivial, so most causally evolving spin network histories are not screen networks. But it is not difficult to construct examples that do satisfy the conditions. For example, for $SU_q(2)$ spin networks we may restrict all labels to spin 1 and all nodes to be four valent, then the splitting may be accomplished with $j_0 = 1$ at all transitions. Finally, we note that since there is no requirement that $\widehat{LR} = \widehat{RL}$ the resulting theory may be chiral.

5 The weak holographic principle

It is not difficult to abstract from the definition of a quantum screen network the main elements of what we propose make a theory holographic, and formulate them as the *weak holographic principle*.

These are:

1. A discrete holographic theory is based on a causal history, that is, the events in the quantum spacetime form a partially ordered set under their causal relations.
2. Among the elements of the quantum spacetime, a set of screens can be identified. Screens are 2-sided objects with two past sides and two future sides.
3. There is a Hilbert space for each past or future side of a screen. Observables on this Hilbert space describe information that an observer at the screen may acquire about the causal past of the screen, by measurements of fields on that side of the immediate past of the screen. There is an algebra of such observables for each side of the screen.
4. Since screens are 2-sided, each has an orientation reversal operation that sends the state space of one side to its complex conjugate on the other side.
5. All observables in the theory are operators in the algebra of observables $\mathcal{A}(s)$ for some screen s .
6. The area of a screen s is either a fixed number a_s , or an operator \hat{A}_s in $\mathcal{A}(s)$. If it is a number, it is proportional to the dimension $D(s)$ of the Hilbert space of either screen side,

$$a_s \propto l_{Planck}^2 \ln D(s). \quad (13)$$

If it is an operator,

$$\mathcal{H}_s = \bigoplus_a \mathcal{H}_s^a \quad (14)$$

where each factor \mathcal{H}_s^a is the eigenspace of $\hat{A}[s]$ with eigenvalue $a[s]$ which each satisfy (13).

6 Conclusions

In this paper, we listed and analyzed the main features that can be expected of a holographic theory of quantum cosmology. Based on this, we stated the holographic principle in a discrete, background independent form.

More can be said about the relationship of the weak holographic principle to its “strong” forms given elsewhere [1, 2, 3, 4]. As we mentioned in the introduction, what needs to be checked is that, when the weak holographic history has a good continuum limit, the strong holographic principle holds in this limit continuum theory. At this stage, little is known about the continuum limit of discrete causal theories like (quantum) screen networks[17, 29]. Ambjorn, Loll, Anagnostopoulos and others have shown that Lorentzian 1 + 1 gravity belongs to a universality class different that Liouville gravity [30, 31]. Similar calculations in higher dimensions are technically very demanding, and it is expected that the results in the causal/Lorentzian case are very different than the euclidean ones.

Requiring that a theory is weakly holographic places constraints on both its dynamics and the algebras of observables. The way it affects the measurement theory that is appropriate in background independent theories of quantum cosmology will be discussed in [33].

Before closing, we briefly consider the possibility that there be a version of the strong holographic principle that may hold at the background independent level. Given the formalism developed here it is possible to state a strong form of the holographic principle for a background independent theory. This makes it possible to identify a problem that would have to be overcome to realize it in the kinds of theories we have considered here.

The strong form of the holographic principle has been stated in certain backgrounds, such as *AdS/CFT* as a conjecture about an equivalence between a boundary theory and a bulk theory[32]. There is no boundary in a cosmological theory, but a screen network such as defined here plays the role of the boundary theory as it describes evolution in terms of a flow of information between Hilbert spaces attached to screens.

A strong form of the holographic principle then requires that there be a bulk theory which has the property that its kinematics and dynamics is exactly equivalent to some screen network theory. Since the theory is expected to be cosmological, so that there is no boundary, the screens are embedded in the bulk. This means that

the bulk theory must have the property that it is equivalent to a different theory that involves only a subset of events which are its screens.

It is easy to imagine that there is a sense in which a sub-history of any history may be defined which gives an approximate description of that history. This could be accomplished by an appropriate coarse-graining. But the strong holographic principle requires more, the screen theory must not just arise from a coarse-graining of the bulk theory, it must be completely equivalent to it.

We can formulate this in the class of theories considered here. For the bulk theory, we consider a general quantum causal history QC . We know from the above that this includes some candidates for background independent quantum theories of gravity. The strong holographic principle would state that for every QC satisfying a certain list of conditions, there is a quantum screen network QS which is equivalent to it. Equivalence requires that the relationship be 1-to-1 so that QC can be recovered from QS .

While this is not impossible, it is not difficult to see what kinds of obstacles would have to be overcome to accomplish it. To do this we consider how the results of this paper may be extended to the case of a general quantum causal history, some of whose events satisfy the conditions to be screens. Let then QC be a quantum causal history as described in [14]. Namely, if \mathcal{C} is the underlying causal set, QC is the functor

$$QC : EC \rightarrow Hilb \tag{15}$$

where EC is the poset of edge-sets of \mathcal{C} , in which two edge-sets are related when they form a complete pair as defined in section 3. We call a causal history complete when there is an *initial* edge-set A_0 such that $Future(A_0) = \mathcal{C}$. An initial state is a choice of $|\Psi\rangle \in H(A_0)$. Given QC , each such initial state determines a density matrix in the Hilbert space of any other edge-set in the causal history. It follows that, if a quantum causal history contains quadruples of events which are screens, a choice of initial state will determine a density matrix in each screen Hilbert space, corresponding to information flowing across the screen.

For the strong holographic principle to hold in the form just stated, two things are required. First, it must be possible to find a screen network containing the subset of events of \mathcal{C} that are screens, and with the property that the density matrix in any screen Hilbert space is fully determined by the density matrices on its past screens. Second, it should be possible to reconstruct QC from QS .

Even if the first can be done, there is a general difficulty with the second. The problem is that there is no natural notion of a quantum causal history which is a subhistory of another. Where one causal set may be a subset of another, the same is not the case for the corresponding quantum causal histories, as the covering relations are singled out in the construction. Since not all covering relations of the screen subset are covering relations in the causal set, there is no natural restriction of the functor QC that reduces the quantum causal history to a quantum causal history on the subset.

This is related to the fact that the flow of information through a quantum causal history is path-dependent, which is also a feature of the flow of information in the semiclassical theory on a general, curved, spacetime. The quantum information that flows between two screens will in general be a superposition of the effects of several evolution operators.

Since there are always fewer screens in S than events in the original \mathcal{C} , the number of covering relations in S are less than those in \mathcal{C} . Thus, it will not, in general, be possible to invert the procedure and use the data in QS to determine a unique quantum causal history QC . Rather, a definition of a subhistory of QC should involve a suitable notion of coarse-graining in which information about the original history is lost. Unless this can be avoided, there will not be a unique bulk history QC which is determined from the screen network history QS .

In either case, what will be true is that the theory will satisfy the weak holographic principle, in the form we have given it here.

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References

- [1] G. 'tHooft, *Dimensional reduction in quantum gravity* gr-qc/9310006.
- [2] L. Susskind, *The world as a hologram*, hep-th/9409089, J. Math. Phys. (1995) ; *Strings, black holes and Lorentz contractions* hep-th/9308139; Phys. Rev. Lett. 71 (1993) 2367; Phys. Rev. D 49 (1994) 6606; D 50 (1994) 2700; L. Susskind and P. Griffin, *Partons and black holes* hep-th/9410306
- [3] W. Fischler, L. Susskind, *Holography and Cosmology*, hep-th/9806039.
- [4] Bousso R, *Holography in General Space-times* , hep-th/9906022, JHEP 9906 (1999) 028; *A covariant entropy conjecture*, hep-th/9905177, JHEP 9907:004,1999.

- [5] Ideas closely related to the holographic principle were presented by L. Crane, *Clocks and Categories, is quantum gravity algebraic?* J.Math.Phys. 36 (1995) 6180-6193, gr-qc/9504038; *Topological field theory as the key to quantum gravity*, hep-th/9308126, in *Knot theory and quantum gravity* ed. J. Baez, —(Oxford University Press); *Categorical Physics*, hep-th/9301061.
- [6] Other discussions of the holographic principle in cosmological contexts include, Nemanja Kaloper, Andrei Linde, hep-th/9904120; Eanna E. Flanagan, Donald Marolf and Robert M. Wald, hep-th/9908070; Richard Dawid, hep-th/9907115; Ram Brustein, Stefano Foffa and Riccardo Sturani, hep-th/9907032; G. Veneziano, hep-th/9907012; Sougato Bose, Anupam Mazumdar, gr-qc/9909008; Dongsu Bak, Soo-Jong Rey, hep-th/9811008; P. A. Zizzi, gr-qc/9907063; Herman Verlinde, hep-th/9906182; K.-H. Rehren, hep-th/9905179; Dongsu Bak, Soo-Jong Rey, hep-th/9902173; Richard Easther, David A. Lowe, hep-th/9902088; Phys.Rev.Lett. 82 (1999) 4967-4970; P. Horava, hep-th/9712130; Phys.Rev. D59 (1999) 046004.
- [7] R. Penrose, *Theory of quantized directions* unpublished manuscript.; in *Quantum theory and beyond* ed T Bastin, Cambridge U Press 1971; in *Advances in Twistor Theory*, ed. L. P. Hughston and R. S. Ward, (Pitman,1979) p. 301; in *Combinatorial Mathematics and its Application* (ed. D. J. A. Welsh) (Academic Press,1971).
- [8] D. Finkelstein, *Spacetime Code 1-6* Phys. Rev. D5:320,1972, D5:2922,1972, Phys.Rev.D9:2219,1974, Phys.Rev.D9:2231,1974
- [9] L. Smolin, *The future of spin networks* in *The Geometric Universe* (1997), Oxford University Press, 1998, ed. S. A. Huggett et al. gr-qc/9702030; in *Quantum Gravity and Cosmology*, eds J Pérez-Mercader *et al*, World Scientific, Singapore 1992.
- [10] C. Rovelli and L. Smolin *Discreteness of area and volume in quantum gravity* Nuclear Physics B 442 (1995) 593. Erratum: Nucl. Phys. B 456 (1995) 734; “Spin networks and quantum gravity” gr-qc/9505006, Physical Review D 52 (1995) 5743-5759.
- [11] C. Rovelli, gr-qc/9710008, also in the electronic journal Living Reviews.
- [12] T. Banks, W. Fishler, S. H. Shenker, L. Susskind, *M theory as a matrix model: a conjecture* hep-th/9610043; Phys. Rev. D55 (1997) 5112; Lubos Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025; Tom Banks, Nathan Seiberg, *Strings from Matrices*, Nucl.Phys. B497 (1997) 41-55, hep-th/9702187 ; R. Dijkgraaf, E. Verlinde, H. Verlinde, *Matrix String Theory*, Nucl.Phys. B500 (1997) 43-61, hep-th/9703030; M Kato: Phys Lett B245 (1990)

43. K Konishi G Paffuti P Provero: Phys Lett B234 (1990) 276. G Veneziano: Europhys Lett 2 (1986)199. D Amati M Ciafaloni G Veneziano Phys. Lett. B216 (1989) 41; R Guida K Konishi P Provero: Mod Phys Lett A6 (1991)1487; J J Atick E Witten, Nucl Phys B310 (1988) 291 ; D Gross and PF Mende, Phys Lett B197 (1987) 129 ; Nucl Phys B303 (1988) 407
- [13] Support for this conclusion can be found in the results of, Reza Tavakol and George Ellis, ON HOLOGRAPHY AND COSMOLOGY, hep-th/9908093; Ted Jacobson, ON THE NATURE OF BLACK HOLE ENTROPY. To be published in the proceedings of 8th Canadian Conference on General Relativity and Relativistic Astrophysics, Montreal, Quebec, Canada, 10-12 Jun 1999, gr-qc/9908031; L. Smolin *Weak vrs strong forms of the holographic principle*, preprint to appear, 1999.
- [14] Markopoulou F, 1999, "Quantum causal histories", hep-th/9904009.
- [15] Markopoulou F, 1998, "The internal logic of causal sets: What the universe looks like from the inside", gr-qc/98011053.
- [16] Markopoulou F, 1997, "Dual formulation of spin network evolution", gr-qc/9704013.
- [17] Markopoulou F and Smolin L, 1997, "Causal evolution of spin networks", *Nucl Phys B* **508** 409.
- [18] Markopoulou F and Smolin L, 1998, "Quantum geometry with intrinsic local causality", *Phys Rev D* **58**, 084032.
- [19] Markopoulou F and Smolin L, 1998, "Nonperturbative dynamics for abstract (p, q) string networks", *Phys Rev D* **58**, 084033.
- [20] L. Smolin, *A candidate for a background independent formulation of M theory*, hep-th/9903166.
- [21] J. D. Bekenstein, Lett. Nuovo Cim 11 (1974).
- [22] Criscuolo A and Waelbroeck H, 1998, "Causal Set Dynamics: A Toy Model", gr-qc/9811088.
- [23] Bombelli L, Lee J, Meyer D and R, 1987, "Space-time as a causal set", *Phys Rev Lett* **59** 521.
- [24] Meyer D A, 1988, *The Dimension of Causal Sets*, PhD Thesis, Massachusetts Institute of Technology.
- [25] Sorkin R, 1990, "Space-time and causal sets" in Proc. of SILARG VII Conf., Coyococ, Mexico.

- [26] G. 't Hooft, *Quantum gravity: a fundamental problem and some radical ideas*. Cargèse Summer School Lectures 1978. Publ. “Recent Developments in Gravitation”. Cargèse 1978. Ed. by M. Lévy and S. Deser. Plenum, New York/London, 323; *Quantization of Space and Time in 3 and in 4 Space-time Dimensions*, Lectures held at the NATO Advanced Study Institute on “Quantum Fields and Quantum Space Time”, Cargèse, July 22 – August 3, 1996. gr-qc/9608037; *The scattering matrix approach for the quantum black hole: an overview*. J. Mod. Phys. **A11** (1996) pp. 4623-4688. gr-qc/9607022.
- [27] Y. Ling and L. Smolin, *Supersymmetric Spin Networks and Quantum Supergravity*, hep-th/9904016, to appear in Physical Review D
- [28] E. Verlinde, *Fusion rules and modular transformations in 2D conformal field theory* Nucl. Phys. B 300 (1988) 360
- [29] S. Kauffman and L. Smolin, *Combinatorial dynamics in quantum gravity* hep-th/9809161, to appear in the Proceedings of the Karpaz 99 winter school, 1999.
- [30] Ambjorn J and Loll R, 1998, “Non-perturbative Lorentzian Quantum Gravity, Causality and Topology Change” *Nucl Phys* **B536** 407.
- [31] Ambjorn J, Nielsen JL, Rolf J and Loll R, 1998, “Euclidean and Lorentzian Quantum Gravity - Lessons from Two Dimensions”, hep-th/9806241.
- [32] Juan M. Maldacena, hep-th/9711200, hep-th/9803002; E. Witten, hep-th/9802150, hep-th/9803131; S.S. Gubser, I.R. Klebanov, A.M. Polyakov, Phys.Lett. B428 (1998) 105-114, hep-th/9802109; for a review, see, O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, Y. Oz, *Large N Field Theories, String Theory and Gravity* hep-th/9905111.
- [33] Markopoulou F and Smolin L, 1999, “Causal quantum cosmology”, to appear.